NUMERICAL STUDY OF THE CEREBRO-SPINAL FLUID (CSF) DYNAMICS UNDER QUASI-STATIC CONDITION DURING A CARDIAC CYCLE

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Abstract - **In this study, we present a method to perform a numerical simulation of the flow dynamics of the Cerebrospinal Fluid (CSF) based on anatomical Magnetic Resonance Images (MRI). The Computational Fluid Dynamics (CFD) software, written in language C, integrates different numerical schemes to solve the governing equations. The time derivatives were discretized using the Crank-Nicolson scheme. The equation of continuity was modified by introducing an artificial compressibility and discretized by a finite difference scheme. The meshed boundary of the CSF was immersed in a Marker-And-Cell staggered grid for to take into account fluid-structures interactions. Equations of hydrodynamics were solved with an iterative method under different quasi-static conditions. The anatomical basis of our simulations was generated from individual MRI scans. The surface of the anatomical flow channels of interest was extracted by segmentation and triangulated. In parallel to the acquisition of the anatomical data CSF flow has been measured by MRI. To characterize a whole cardiac cycle sixteen equidistant velocity measurements have been performed. In addition, a home made software was implemented to visualize computed data (velocities, pressure).**

Keywords **– CSF flow; Computational Fluid Dynamics; Magnetic Resonance Imaging; Intra-cranial dynamics; CSF-Brain interactions.**

I. INTRODUCTION

The aqueduct of Sylvius is the narrow channel connecting the third and fourth ventricle (Fig 1). It is a narrow pathway for the Cerebrospinal Fluid (CSF) flow and hence may be a privileged site for development of hydrocephalus.

The geometry of the aqueduct of Sylvius has often been studied [1,2] and numerous models of the physics of hydrocephalus have been developed [3]. Jacobson [4] realized a numerical simulation of the CSF with a commercial Computational Fluid Dynamics (CFD) software that couldn't take into account elastic and compressive properties of the surrounding brain tissue.

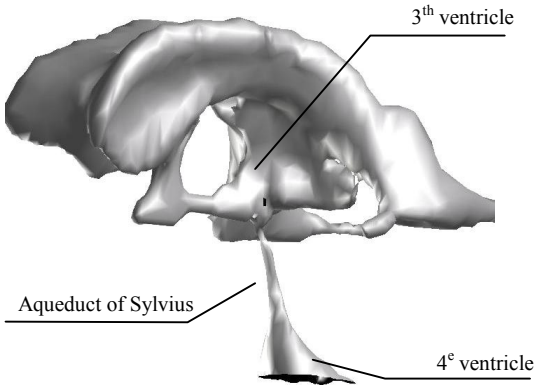

Fig. 1 : three dimensional representation of intra-cerebral CSF spaces.

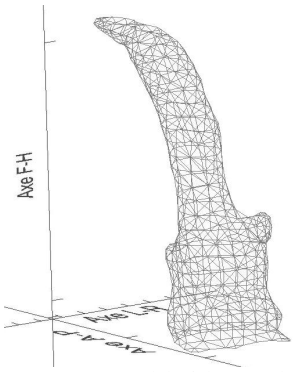

Fig. 2 : 3D model of the aqueduct of Sylvius. The meshing is made of 458 vertices and 881 triangles.

The aim of this study is to realize a realistic numerical simulation of the CSF flow including fluid-structure interactions.

II. METHODOLOGY

A. Anatomy modeling

The first part of this work comprises the generation of a numerical model of the brain-CSF interface (called parenchyma) in which the flow is simulated. Three dimensional T2-weighted MR Images were acquired on a 1.5 T Signa scanner (GEMS, Milwaukee) to provide a good contrast between brain structures and CSF. A software was realized using IDL 5.4 (Interactive Data Language, RSInc France) in order to process image data. The desired CSF spaces were isolated from surrounding tissues in three dimensions by virtual cutting of the upper and lower end and subsequent application of a region growing based segmentation algorithm. This structure was meshed by triangulation and the resulting meshing was organized to fully describe the relationships between triangles, vertices and edges.

B. Numerical simulation

The CFD code was written in ANSI C language based on the work of M. Fluck [5]. Under normal physiological conditions, the CSF behaves as an incompressible Newtonian fluid in a laminar flow [6]. Density and viscosity were $\rho = 10^3$ kg.m^{-3} and $\mu = 10^{-1}$ Pa.s respectively [4].

Under these conditions, the governing laws of hydrodynamics are the equation of continuity :

$$\vec{\nabla}.\vec{u} = 0 \qquad (1)$$

and the Navier-Stokes equation :

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Abstract

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$$\rho\frac{\partial\vec{u}}{\partial t}+\rho(\vec{u}.\vec{\nabla})\vec{u}=-\vec{\nabla}p+\mu\Delta\vec{u}+\vec{f} \qquad (2)$$

where $\vec{u}$ represents the velocity vector (whose components are *u*, *v* and *w* for the x, y and z directions, respectively), *p* the pressure and $\vec{f}$ the forces that act on the fluid.

Different numerical schemes have been used to solve these equations. For the linearization of temporal terms of the Navier-Stokes equations, the *Crank-Nicolson* scheme was chosen for to provide both a good convergence and acceptable truncation errors. The *Artificial Compressibility Method* was applied with the equation of continuity and then linearized with a backward difference scheme.

This system of equations was solved for a finite number of points. The discretization of the spatial domain was realized with the multi-grid *Immersed Boundary Method*. For this method [7], the numerical model of the parenchyma was immersed in a three dimensional Cartesian grid. Grid points were labeled 'Inner' or 'Outer' depending on their location inside or outside the aqueduct. Then, the flow was calculated for inner points. For each point of the grid, velocity components and pressure were staggered (Fig. 3) following the *Marker-And-Cell* (MAC) method.

After the linearization step of relations (1) and (2), an iterative method was used to compute a numerical solution for each grid point from the following algebraic equation system whose unknowns were velocity $\mathbf{u}_{(u,v,w)}$ and pressure *p* :

$$u_l^{m+1}-u_l^m+\varkappa.L_{ui}\left(u_l^m,v_l^m,w_l^m,p_l^m\right)=0 \qquad (3)$$
$$p_l^{m+1}-p_l^m+\lambda.D\left(u_l^m,v_l^m,w_l^m\right)=0$$

where *m* represents the iteration index and *l* the position of the point in the MAC grid. In this expression, the linearized equations are written as functions : $\mathbf{L_{ui}}$ stands for Navier-Stokes for the component number *i* (*i* = 1, 2, 3) of velocity; and **D** stands for the divergence operator in the equation of continuity. λ and κ are two parameters that have to satisfy the following conditions to provide the convergence of the iterative method :

$$0<\boldsymbol{\kappa}\le(\boldsymbol{\Delta x})^2\cdot\left[\frac{2}{Re}+\frac{(\boldsymbol{\Delta x})^2}{2\Delta t}\right]^{-1} \qquad (4)$$
$$0<\boldsymbol{\lambda}\le\frac{(\boldsymbol{\Delta x})^2}{2\boldsymbol{\kappa}}-\left[\frac{1}{\mathbf{Re}}+\frac{(\boldsymbol{\Delta x})^2}{4\Delta t}\right]$$

Re is here the Reynolds number, Δt the time step and Δx the grid spacing..

A correct solution was obtained for iteration index *m+1* if :

$$\left|u_l^{m+1}-u_l^m\right|<\varepsilon,\qquad \varepsilon<<1 \qquad (5)$$

for each of the variables *u*, *v*, *w* and *p*.

Following the *Immersed Boundary Method* proposed by Peskin [7], the coupling between flow and membrane deformation was insured by a three dimensional Dirac delta function. Membrane forces were calculated with the Finite Element Method on each vertex of the meshed surface. The membrane had to mimic the influence of the brain tissues surrounding the aqueduct. Consequently, an approximation has been made and the mechanical properties taken for the membrane were values the of Young's Modulus E and Poisson ratio ν found in the literature [3,8] for the brain supposed homogeneous and visco-elastic (cf. table 1). However, the software imposed the membrane to be rigid in a first validation step.

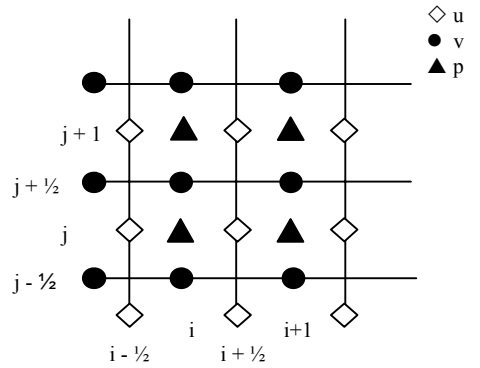

Fig. 3 : 2D staggered arrangement of velocity components and pressure.

TABLE 1
MECHANICAL CONSTANTS FOR DIFFERENT MATERIAL.

Material	Young's Modulus E ($N.m^{-2}$)	Poisson ratio ν	reference
Gray Matter	50	0.49	[8]
White Matter	5	0.49	[8]
"homogeneous Brain"	10	0.35	[3]

	Density □ ($kg.m^{-3}$)	Viscosity □ (Pa.s)	
CSF	10^3	10^{-1}	[4]

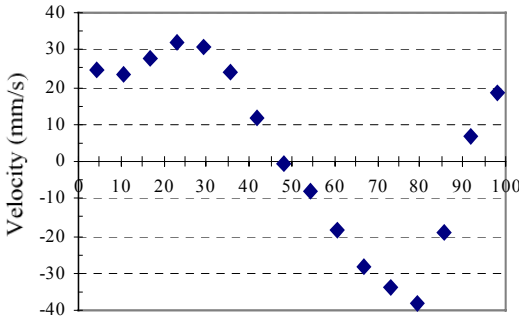

Fig. 4 : distribution of the CSF velocities over a Cardiac Cycle (CC).
Positive values mean downward (caudal) direction while negative mean upward (cranial) direction.

C. Quasi-static flow

The simulation was performed under quasi-static conditions over a cardiac cycle, i.e. the input of the simulations have been the velocities acquired by measurements of CSF flow made by Phase Contrast MR images over a cardiac cycle [9]. "Input" velocities are the boundary conditions imposed on top of the model. Sixteen simulations have been run. For each run, the input velocity profile was chosen to be uniform to reflect the constriction between the 3rd ventricle and the aqueduct.

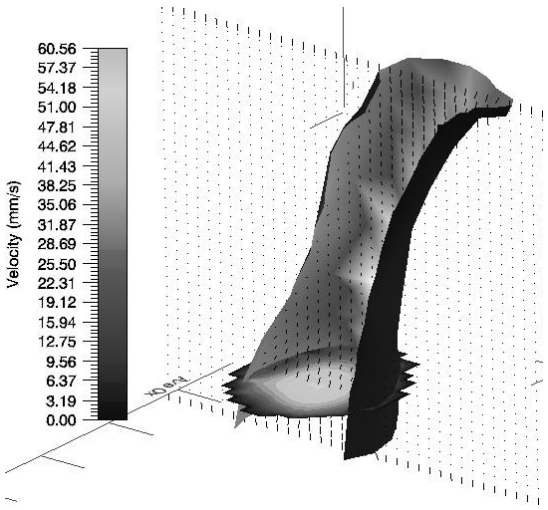

Fig. 5 : Velocity norm (iso-contour) and vector field (black lines) along the aqueduct at about 80% of the Cardiac Cycle.

Further software based on IDL has been developped to analyse and visualise results of the numerical simulation for both flow velocities and pressures.

III. Results and Discussion

Fig. 1 and 2 show the segmented structures and the meshing resulting from the image processing step, respectively. This step allows the representation of thin anatomical structures even with complex geometry. The quality of the model depends on image resolution. With a resolution of about 0.7 mm, a sufficient geometry was computed.

As the Fig. 5 shows, velocities develop along the aqueduct in a parabolic profile. Computed pressures (Fig. 6) are in good agreement with CSF dynamics [9], i.e. at 80% of the Cardiac Cycle, lower pressure in the 3rd ventricle allow the "fill flow" to develop.

Different approximations have been made concerning the brain homogeneity, isotropy and visco-elasticity. These hypotheses are rather coarse, but led to a simpler implementation of the program. No flux through the membrane has been allowed. A porosity term in the model of the brain, as proposed by Tada et al. [8], was omitted.

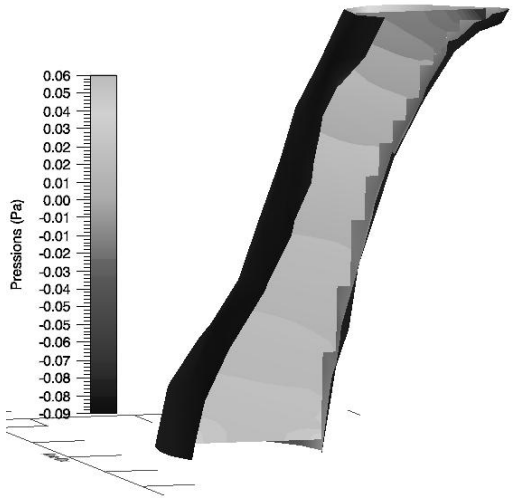

Fig. 6 : Pressure distribution along the aqueduct at about 80% of the Cardiac Cycle.

IV. Conclusion

The presented method allows to simulate interactions between brain and CSF flow based on anatomical data.

This study is the first step toward a fully dynamic simulation with moving boundaries. Further implementation will include calculation of strain in the membrane and, therefore, allow a better description of the intra-cranial physics.

Acquisition of CSF velocities in hydrocephalic patients and individual modeling and simulation with these data may improve our understanding of the pathogenic mechanisms of hydrocephalus development. Moreover, influence of geometry (e.g. partial or total aqueduct obstruction) or brain mechanical properties on hydrocephalus may be another point of interest.

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